第13章 3D图像分析

中国科学技术大学
电子工程与信息科学系

主讲教师：李厚强 (lihq@ustc.edu.cn)
周文罡 (zhwg@ustc.edu.cn)
李　礼 (lil1@ustc.edu.cn)
胡　洋 (eeyhu@ustc.edu.cn)

3D图像分析

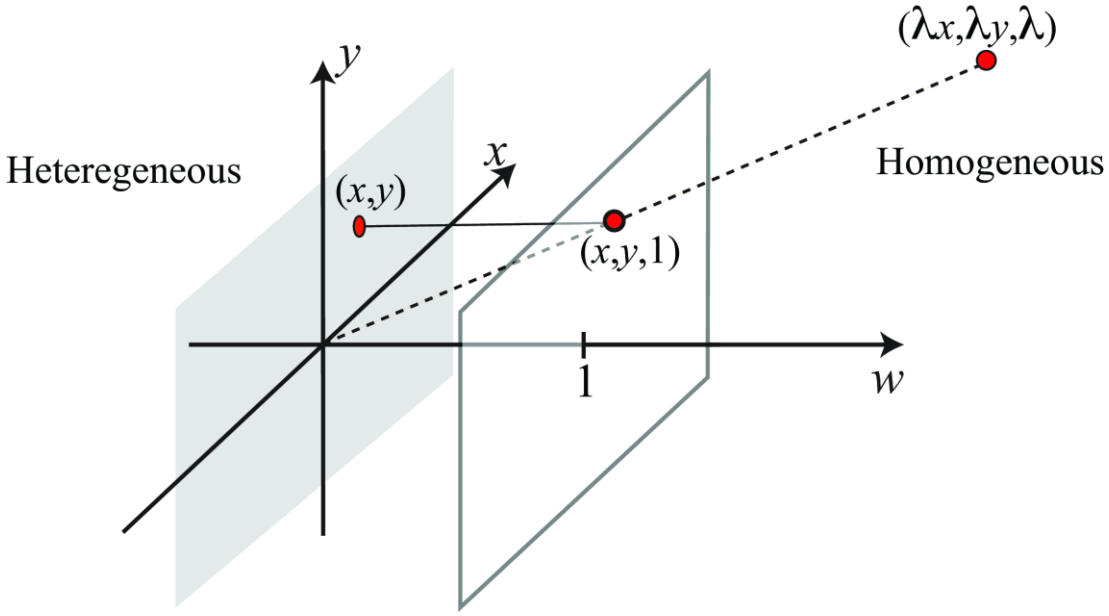
- 传统的三维重建方法
  - 成像变换和相机标定
  - 立体视觉和对极几何
  - 单应性
  - 运动推断结构
- 基于深度学习的三维重建
  - 基于学习的单目深度估计
  - 基于体素的三维表示
  - 基于点云的三维表示
  - 基于多边形网格的三维表示
  - 基于隐函数的三维表示

成像变换

齐次坐标

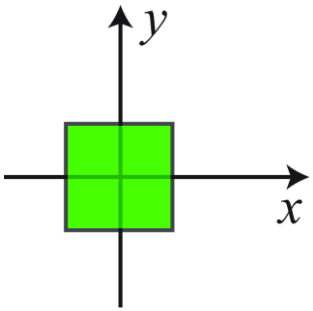
- 笛卡尔坐标 ↔ 齐次坐标

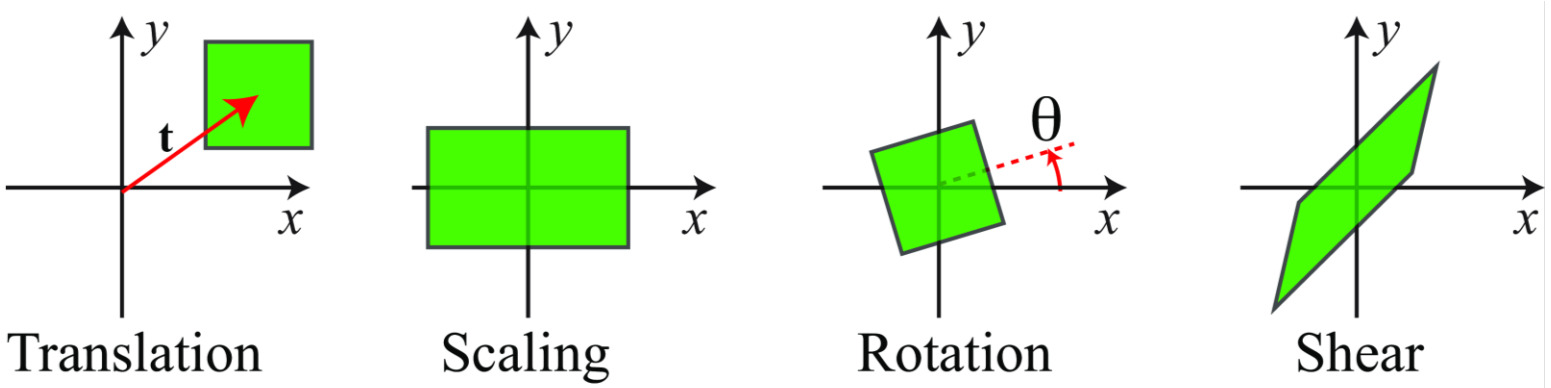
$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ w \end{bmatrix} \rightarrow \begin{pmatrix} x/w \\ y/w \end{pmatrix}$$



成像变换

基本坐标变换



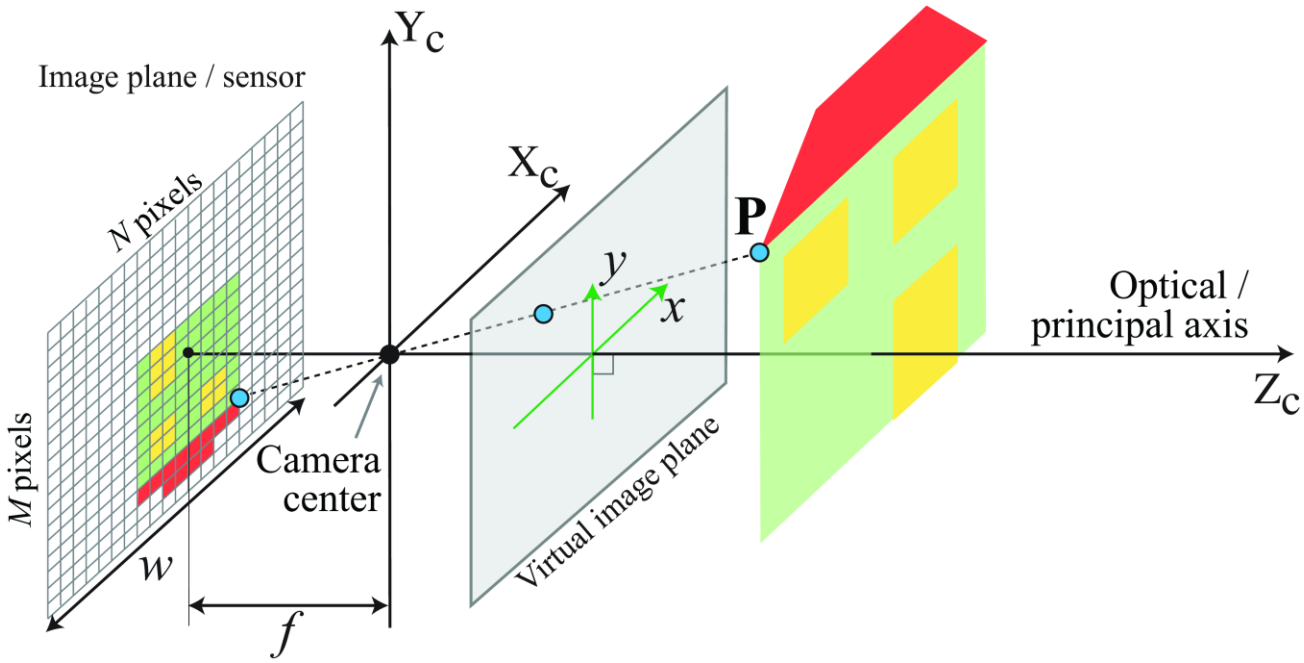


$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & q_x & 0 \\ q_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

成像变换

相机成像模型

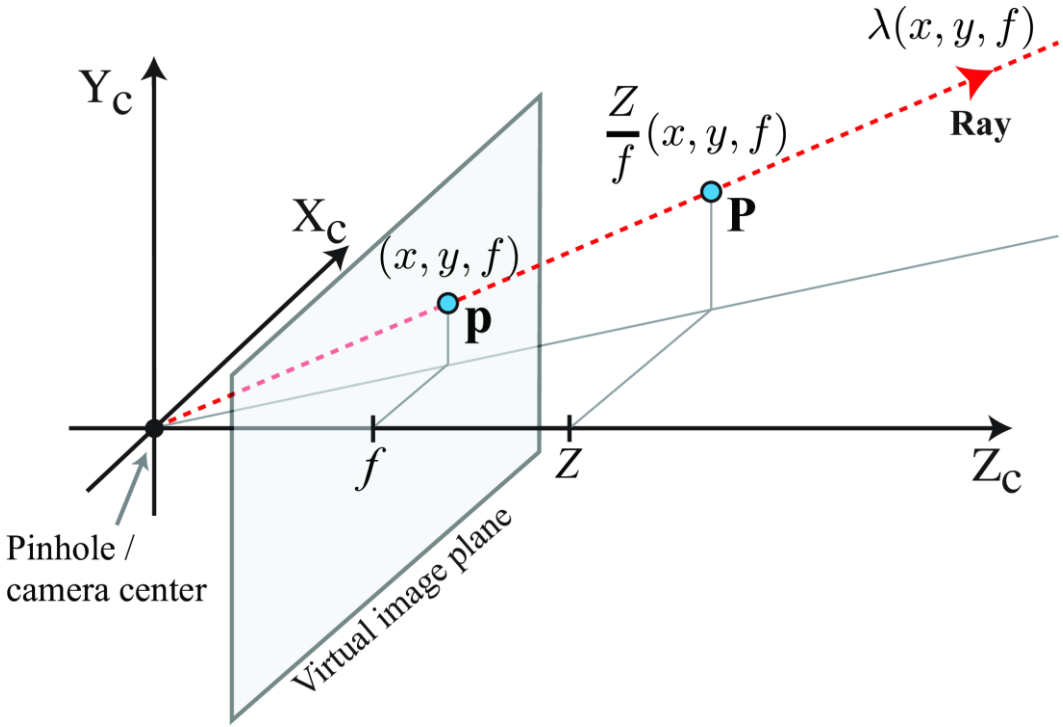


$$\begin{bmatrix} a & 0 & c_x & 0 \\ 0 & b & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \rightarrow \begin{pmatrix} aX/Z + c_x \\ bY/Z + c_y \end{pmatrix} = \begin{pmatrix} n \\ m \end{pmatrix} \qquad a = fN/w$$

相机内参矩阵

成像变换

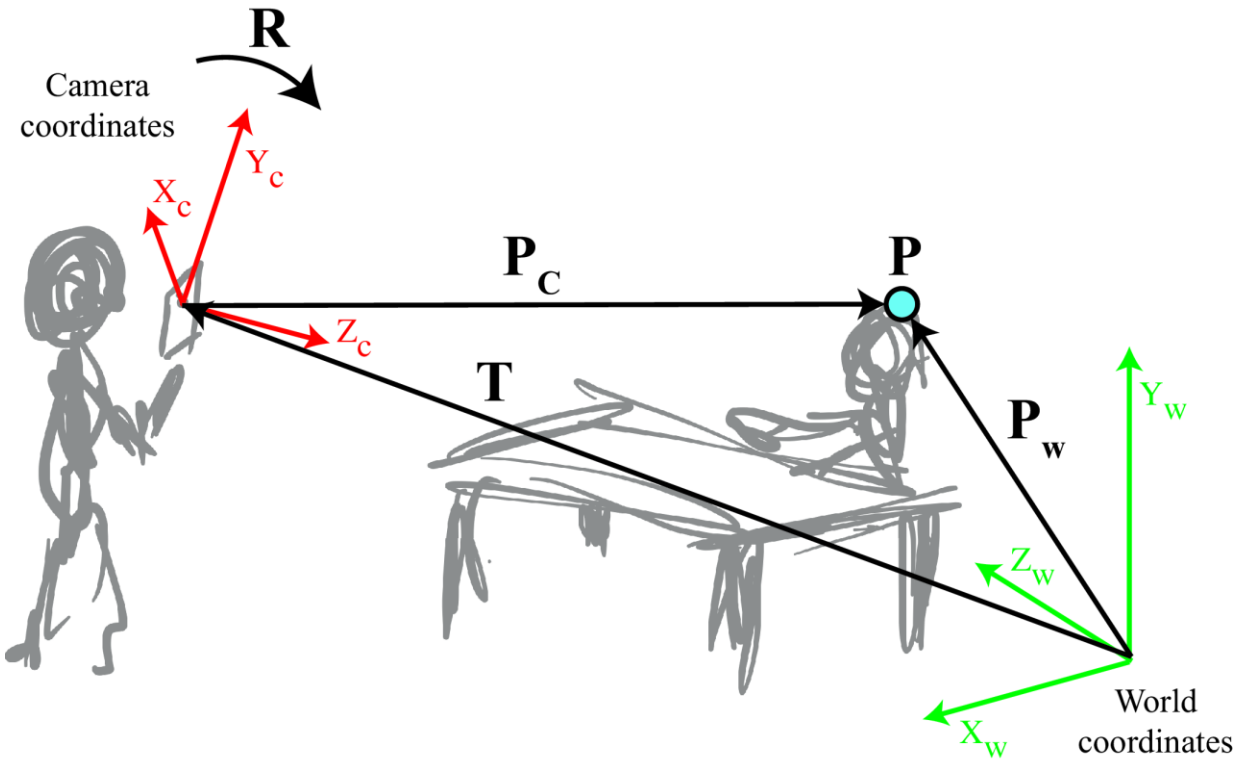
- 逆投影 （从像素到射线）



$$\frac{Z}{f}\begin{pmatrix} x \\ y \\ f \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

成像变换

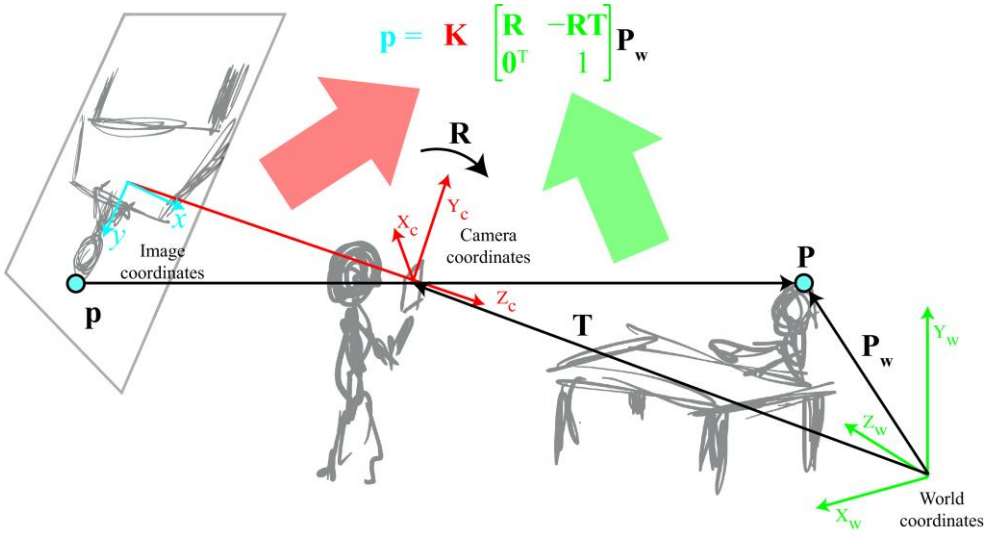
- 相机外参矩阵：相机坐标和世界坐标不在同一位置



$$\mathbf{P}_C = \mathbf{R}(\mathbf{P}_W - \mathbf{T}) = \mathbf{R}\mathbf{P}_W - \mathbf{R}\mathbf{T} \quad 或 \quad \mathbf{P}_C = \begin{bmatrix} \mathbf{R} & -\mathbf{R}\mathbf{T} \\ \mathbf{0}^{\mathsf{T}} & 1 \end{bmatrix} \mathbf{P}_W$$

成像变换

完整成像变换（结合外参和内参矩阵）



$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & 0 & c_x \\ 0 & b & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_1 & R_2 & R_3 \\ R_4 & R_5 & R_6 \\ R_7 & R_8 & R_9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -T_X \\ 0 & 1 & 0 & -T_Y \\ 0 & 0 & 1 & -T_Z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

K是上三角矩阵；
R是正交矩阵

或 $\mathbf{p} = \mathbf{K}[\mathbf{R} \mid -\mathbf{RT}]\mathbf{P}_W$ $\mathbf{M} = \mathbf{K}\begin{bmatrix} \mathbf{R} & -\mathbf{RT} \\ \mathbf{0}^T & 1 \end{bmatrix}$

相机矩阵

相机标定

DLT(Direct Linear Transform)算法

- 对像素 $\mathbf{p}_i$ 和其对应的3D点 $\mathbf{P}_i$

$$\mathbf{p}_i = \mathbf{M}\mathbf{P}_i = \begin{bmatrix} \mathbf{m}_0^\top \\ \mathbf{m}_1^\top \\ \mathbf{m}_2^\top \end{bmatrix} \mathbf{P}_i \qquad \text{即} \qquad x_i = \frac{\mathbf{m}_0^\top \mathbf{P}_i}{\mathbf{m}_2^\top \mathbf{P}_i}, \quad y_i = \frac{\mathbf{m}_1^\top \mathbf{P}_i}{\mathbf{m}_2^\top \mathbf{P}_i}$$

整理成矩阵形式

$$\begin{bmatrix} -\mathbf{P}_i^\top & \mathbf{0} & x_i\mathbf{P}_i^\top \\ \mathbf{0} & -\mathbf{P}_i^\top & y_i\mathbf{P}_i^\top \end{bmatrix} \begin{bmatrix} \mathbf{m}_0 \\ \mathbf{m}_1 \\ \mathbf{m}_2 \end{bmatrix} = \mathbf{0}$$

- 对N对点

$$\begin{bmatrix} -\mathbf{P}_1^\top & \mathbf{0} & x_1\mathbf{P}_1^\top \\ \mathbf{0} & -\mathbf{P}_1^\top & y_1\mathbf{P}_1^\top \\ \vdots & \vdots & \vdots \\ -\mathbf{P}_N^\top & \mathbf{0} & x_N\mathbf{P}_N^\top \\ \mathbf{0} & -\mathbf{P}_N^\top & y_N\mathbf{P}_N^\top \end{bmatrix} \begin{bmatrix} \mathbf{m}_0 \\ \mathbf{m}_1 \\ \mathbf{m}_2 \end{bmatrix} = \mathbf{0}$$

可解得M

相机标定

由M恢复相机内参和外参

$$\mathbf{M} = [\mathbf{KR} \mid -\mathbf{KRT}]$$

- 计算T

  设 $\mathbf{M} = [\mathbf{B}\ \mathbf{b}]$ ， $\mathbf{B} = \mathbf{KR}$ ， $\mathbf{b} = -\mathbf{KRT}$,则

$$\mathbf{T} = -\mathbf{B}^{-1}\mathbf{b}$$

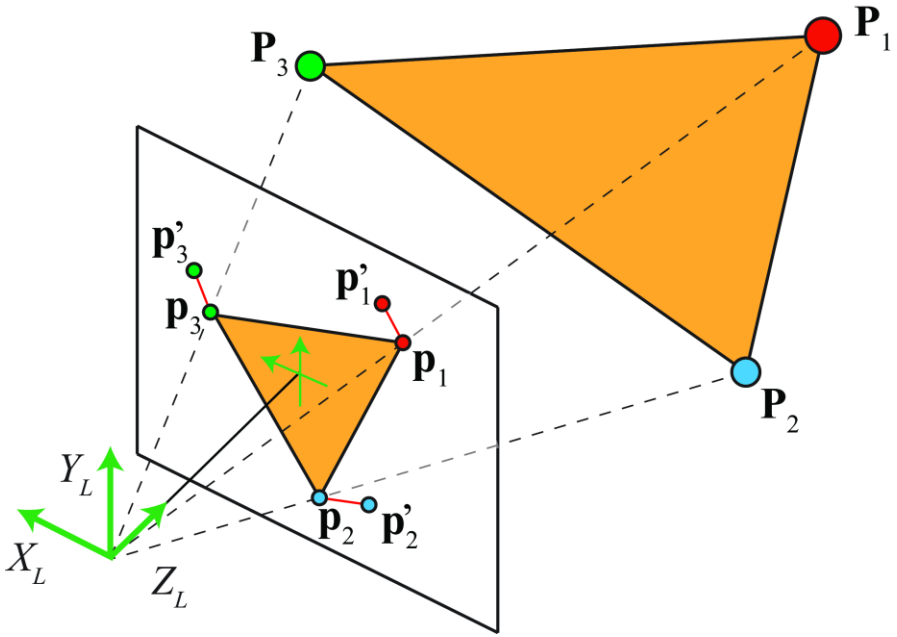
- 计算K, R

  由于K为上三角矩阵，R为正交矩阵，通过对B^{-1}进行QR分解可得到对K, R的估计

- 此估计算法对噪声较敏感，但得到的相机参数可作为其他标定算法的初始值

相机标定

最小重投影误差法



$$\sum_{i=1}^{N} \|\mathbf{p}_i - \mathbf{p}'_i\|^2 = \sum_{i=1}^{N} \|\mathbf{p}_i - \pi(\mathbf{K}[\mathbf{R} \mid -\mathbf{RT}]\mathbf{P}_i)\|^2$$

Π函数将齐次坐标变为笛卡尔坐标

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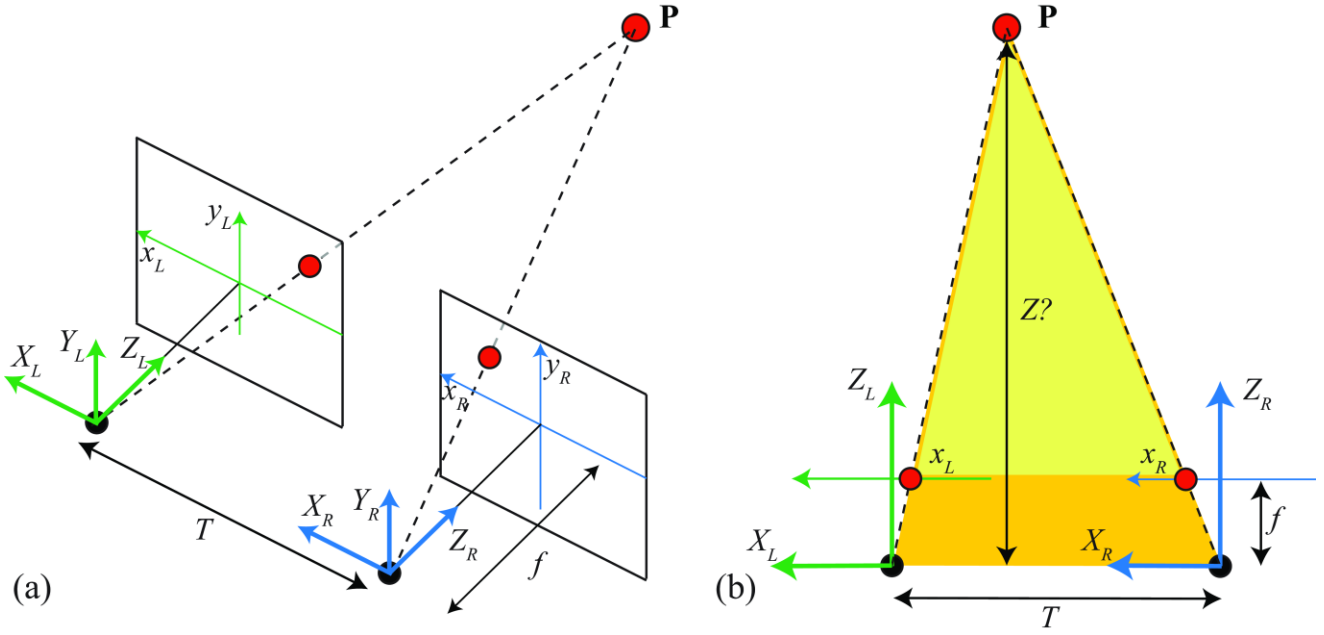
立体视觉

- 立体视觉原理 （both cameras are identical (same intrinsic parameters), and one is translated horizontally with respect to the other）

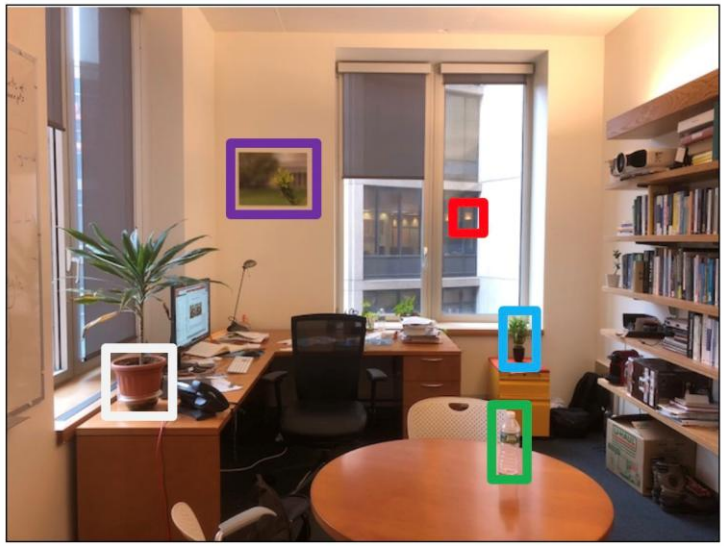


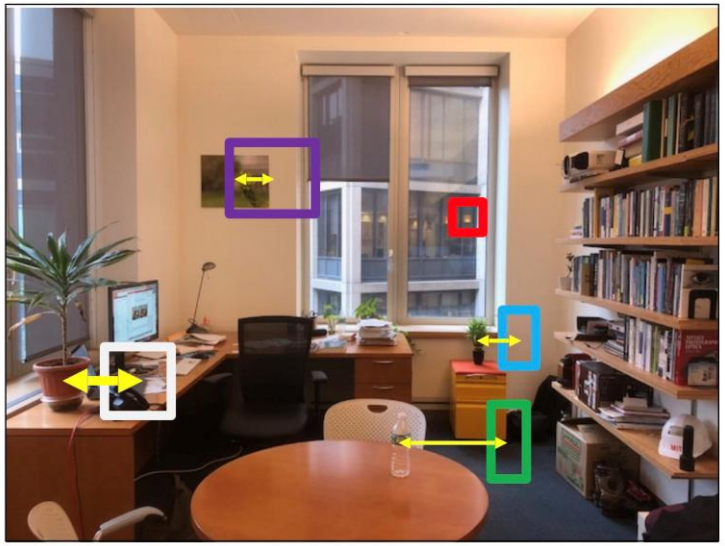
$$\frac{T + x_L - x_R}{Z - f} = \frac{T}{Z} \quad \text{即} Z = \frac{fT}{x_R - x_L}$$

视差（Disparity）

立体视觉

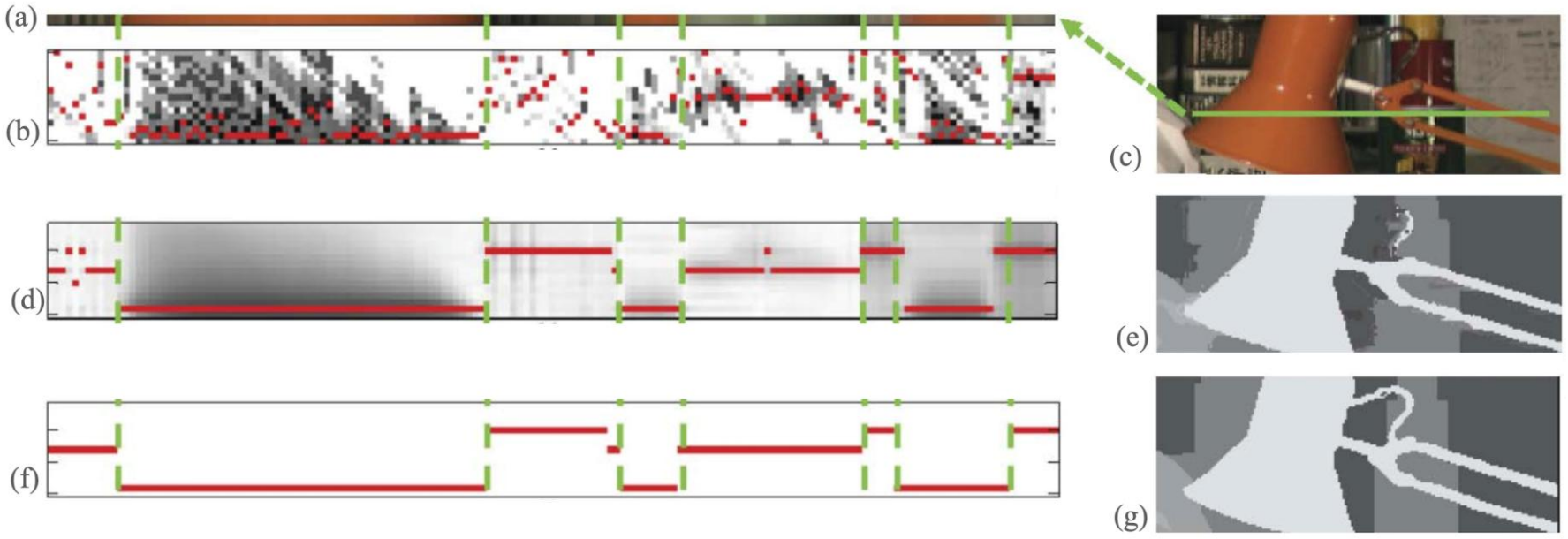
- 立体匹配
  - 不清楚两个视频的对应点





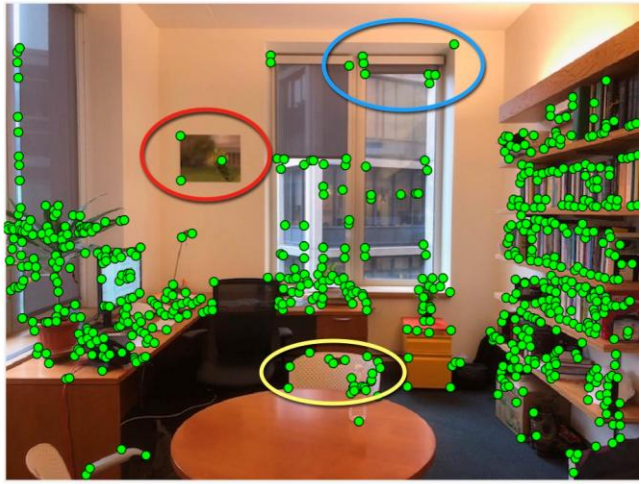
立体视觉

□ 基于亮度的匹配



立体视觉

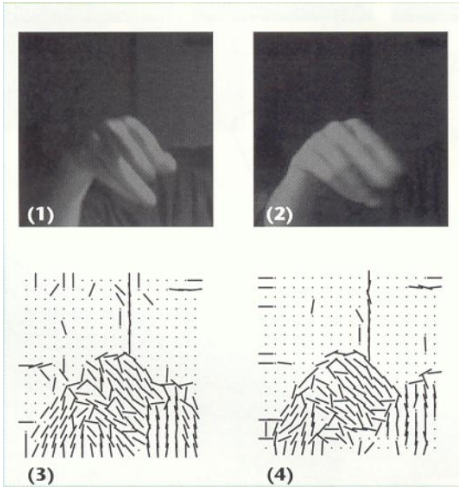
- 基于局部特征的匹配
  - 关键点检测 （比如Harris角点）

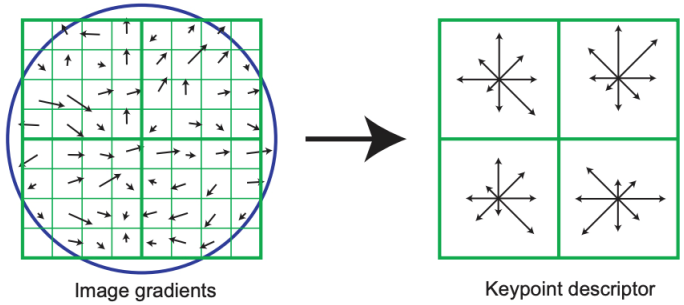


立体视觉

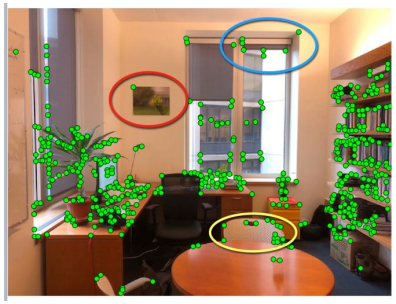
基于局部特征的匹配

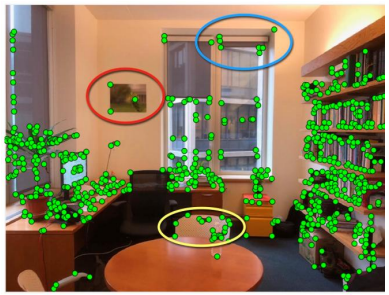
- 局部图像描述 （SIFT）

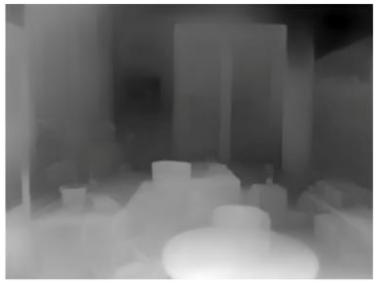




- 深度插值

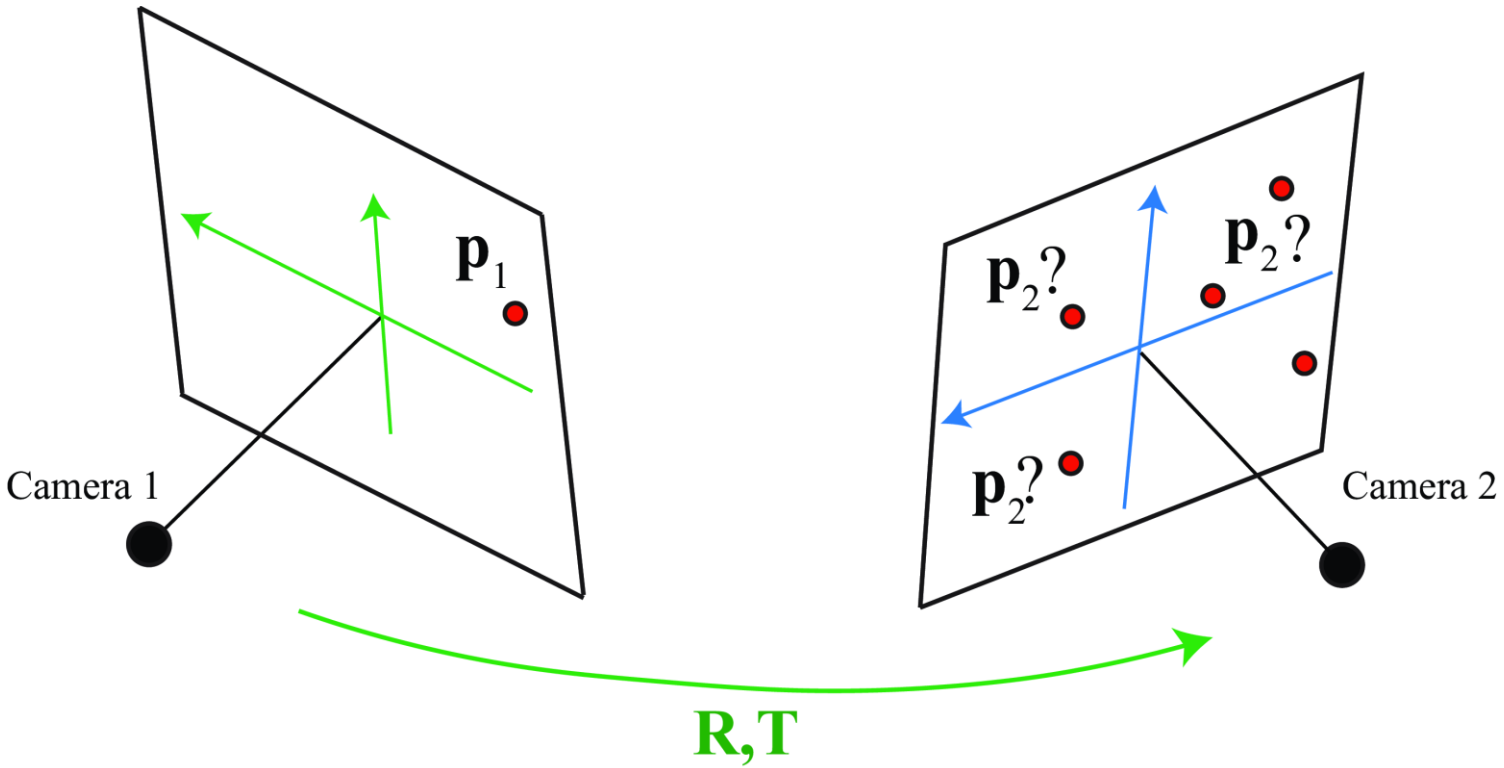






对极几何

- 描述两个不同视图下，一对匹配点之间的几何约束关系（两个视角不在一个水平线的情况）



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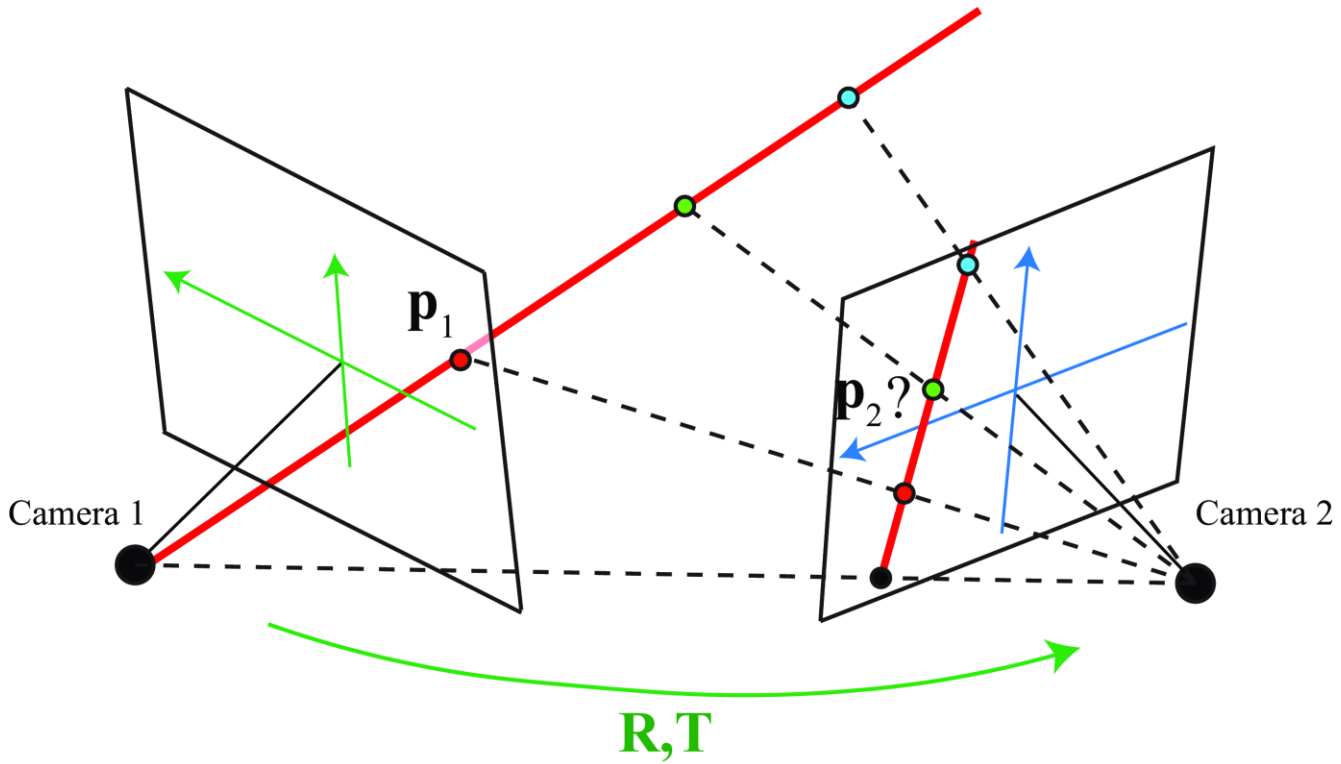
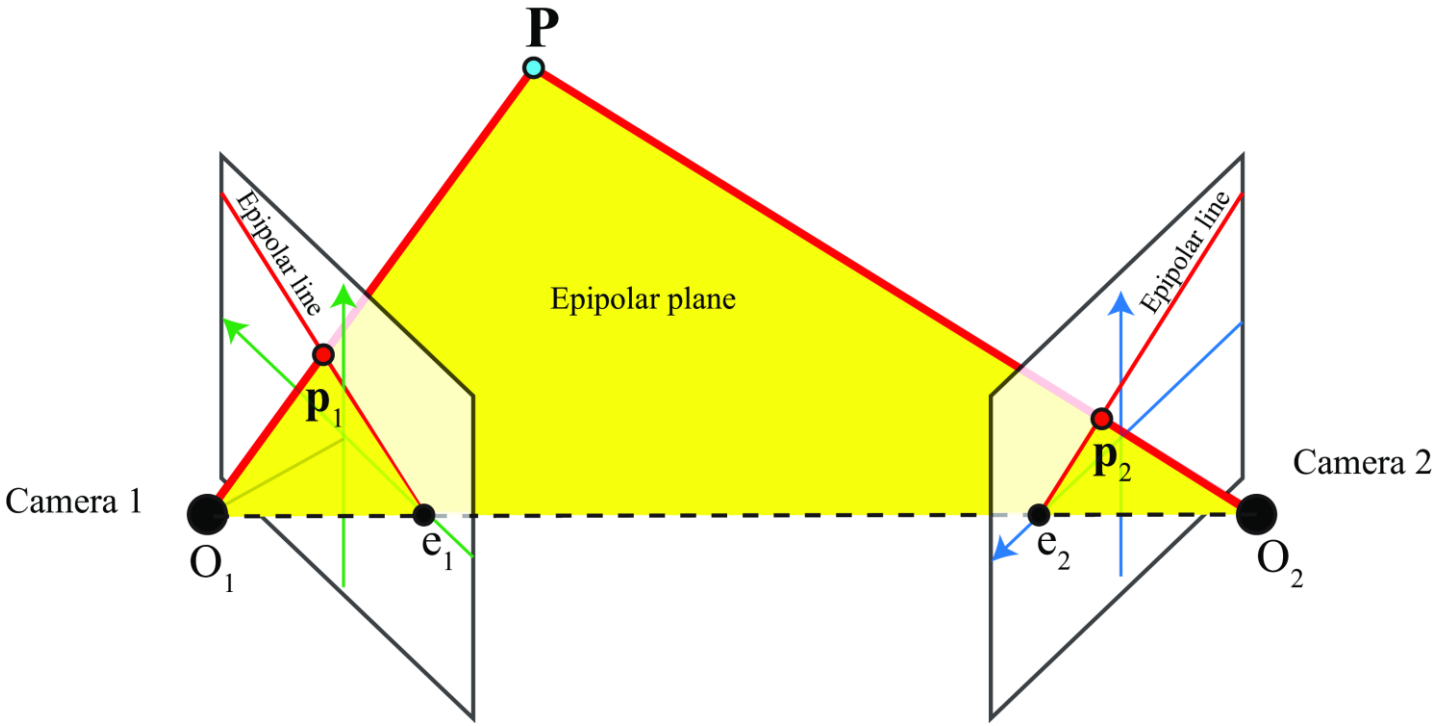


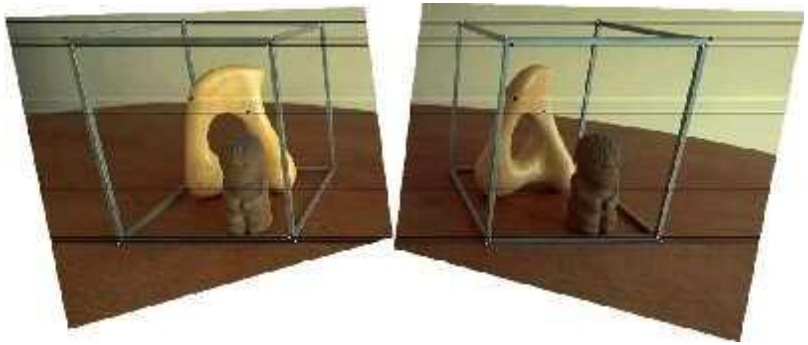
对极几何

- 描述两个不同视图下，一对匹配点之间的几何约束关系
  - Epipolar plane
  - Epipolar line
  - Epipoles



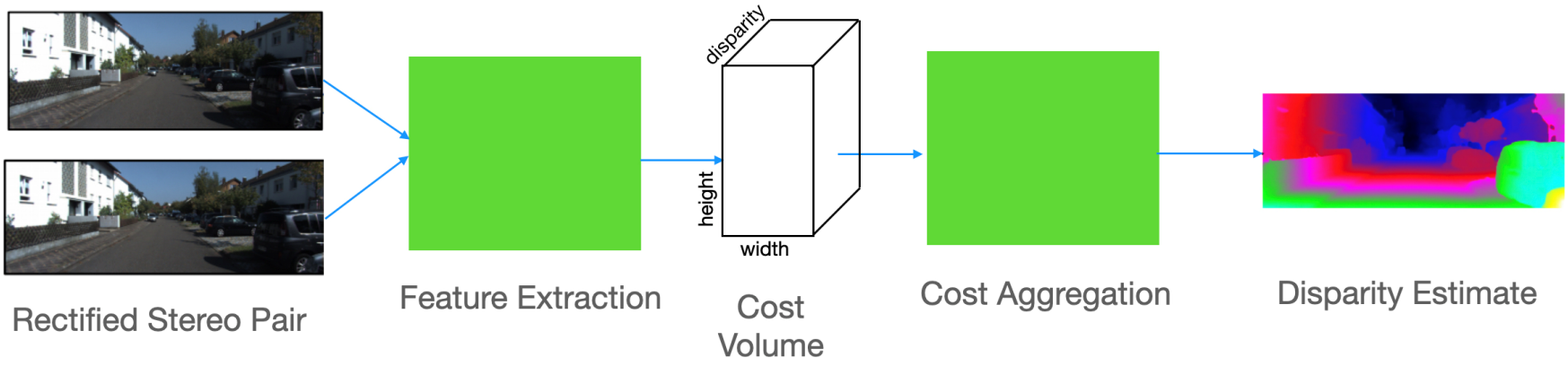
图像矫正(Image Rectification)

- 对任意视角获取的两张图，通过对相机进行旋转、并对图像进行缩放等变换，使得两张图像位于同一水平高度且相互平行
- 矫正后，两张图像上的对应点位于具有相同高度点水平扫描线上，从而可简化特征匹配



基于学习的方法

- 基于学习的深度估计



$$J(\theta) = \sum_t \left| \mathbf{d} - f_\theta(\mathbf{v}_1, \mathbf{v}_2) \right|^2$$

$$f(\mathbf{v}_1, \mathbf{v}_2) = c\left(g(\mathbf{v}_1), g(\mathbf{v}_2)\right)$$

J.-R. Chang, Y.-S. Chen, Pyramid stereo matching network., in: Proceedings of the IEEE/CVF Computer Vision and Pattern Recognition, 2018.

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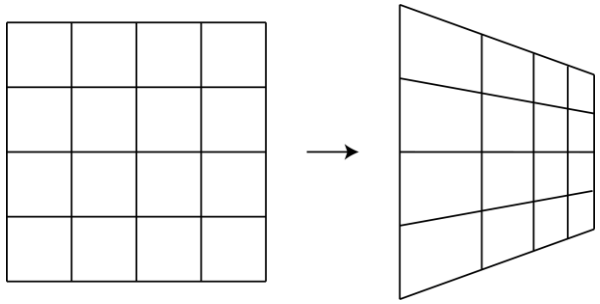
单应性(Homography)

- 单应性变换：点变换函数$\mathrm{p}' = h(\mathrm{p})$，若点$\mathrm{p}_1$，$\mathrm{p}_2, \mathrm{p}_3$在一条直线上，则变换后的点$h(\mathrm{p}_1), h(\mathrm{p}_2), h(\mathrm{p}_3)$也共线



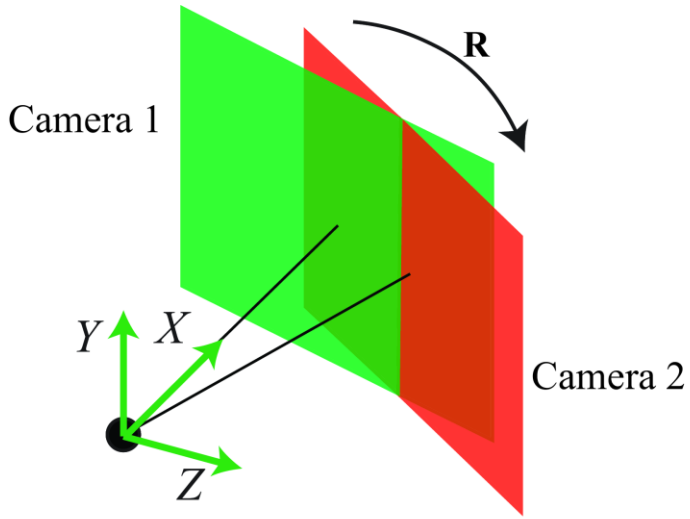
  - 用齐次坐标可表示为

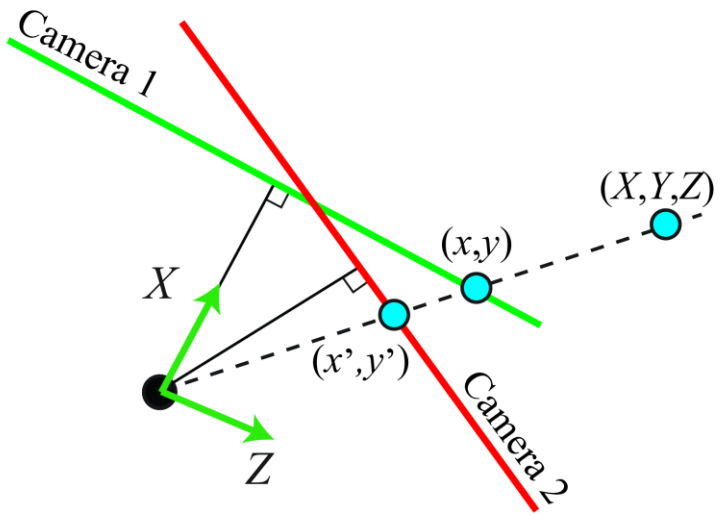
$$\mathbf{p}' = \mathbf{H}\mathbf{p}$$

  - 3×3矩阵H为单应矩阵，8个自由度
  - 共线证明：对直线 $l^{\mathrm{T}}\mathbf{p} = 0$，又 $\mathbf{p} = \mathbf{H}^{-1}\mathbf{p}'$，则有$(\mathbf{H}^{-\mathrm{T}}l)^{\mathrm{T}}\mathbf{p}' = 0$，即变换后仍然为直线

单应性(Homography)

- 相机仅旋转，图像存在单应性





$$\lambda_1 \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathbf{K}[\mathbf{I} \quad \mathbf{0}] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\lambda_2 \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{K}[\mathbf{R} \quad \mathbf{0}] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{KR} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

单应性(Homography)

- 相机仅旋转，图像存在单应性

$$\lambda_1 \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathbf{K}[\mathbf{I} \quad \mathbf{0}] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \qquad \lambda_2 \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{K}[\mathbf{R} \quad \mathbf{0}] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{KR} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

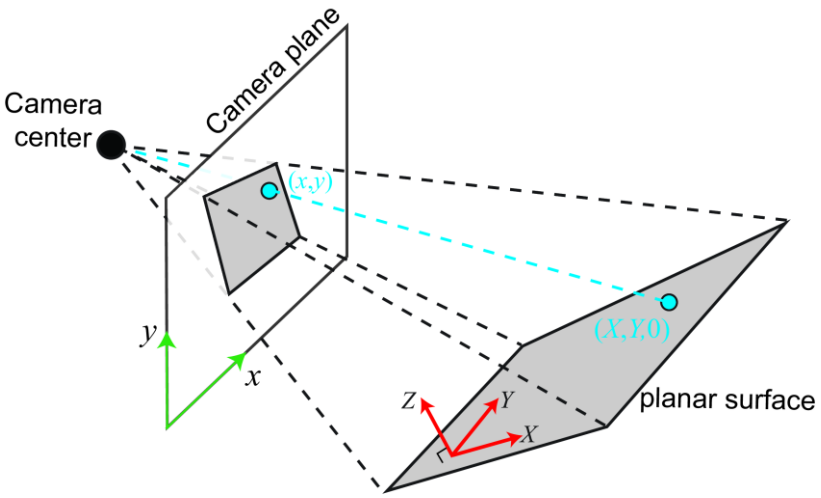
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \lambda_1 \mathbf{K}^{-1} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \lambda_2 \mathbf{R}^{-1} \mathbf{K}^{-1} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

$$\lambda_2 / \lambda_1 \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{KRK}^{-1} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

即 $$\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

单应性(Homography)

- 平面物体图像存在单应性



$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}] \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} = \mathbf{K}[\mathbf{c}_1 \quad \mathbf{c}_2 \quad \mathbf{t}] \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

即 $$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

单应性(Homography)

- 应用举例：全景图像拼接
  - 仅有摄像头转动得到的不同视角的图像完成拼接

单应性(Homography)

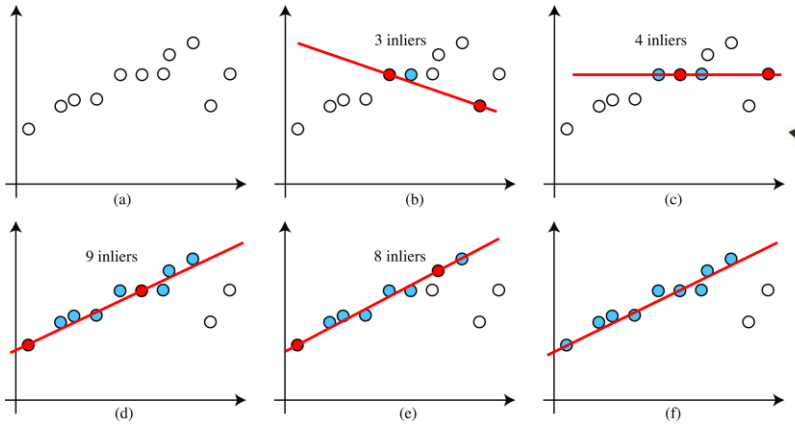
应用举例：全景图像拼接

- DLT算法

$$\mathbf{Ah} = \mathbf{0}$$

  - ✓ $\mathbf{A}$ 为 $2N \times 9$ 矩阵，$\mathbf{h}$ 为 9×1向量
  - ✓ 最小化 $\|\mathbf{Ah}\|^2$，最优解为$\mathbf{A}$的最小奇异值对应的奇异向量，或 $\mathbf{A}^T\mathbf{A}$的最小特征值对应的特征向量

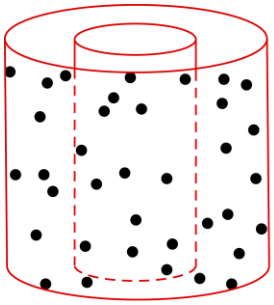
- RANSAC



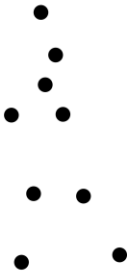
3D图像分析

- 传统的三维重建方法
  - 成像变换和相机标定
  - 立体视觉和对极几何
  - 单应性
  - 运动推断结构
- 基于深度学习的三维重建
  - 基于学习的单目深度估计
  - 基于体素的三维表示
  - 基于点云的三维表示
  - 基于多边形网格的三维表示
  - 基于隐函数的三维表示

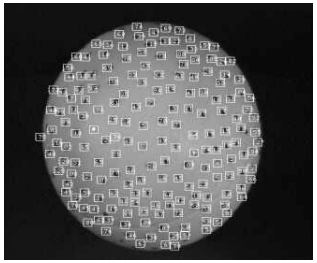
运动推断结构(Structure from Motion)



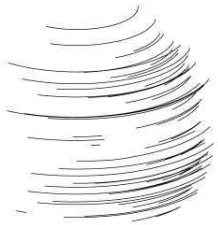
(a)



(b)



(a)



(b)

(c)

Photo Tourism

Exploring photo collections in 3D

Microsoft

(a)

(b)

(c)

运动推断结构(Structure from Motion)

问题定义

- 假设场景中有N个不同的3D点$\mathbf{P}_i$，有M个不同的视角
- 对每个视角，相机参数为$\mathbf{K}^{(j)}, \mathbf{R}^{(j)}, \mathbf{T}^{(j)}$
- 对每个视角，点$\mathbf{P}_i$的投影为$\mathbf{p}_i^{(j)}$
- SFM任务定义为给定一系列有对应关系的2D点$\mathbf{p}_i^{(j)}$，恢复其对应的3D点$\mathbf{P}_i$，及M组相机参数$\mathbf{K}^{(j)}, \mathbf{R}^{(j)}, \mathbf{T}^{(j)}$

分析

- 设相机内参$\mathbf{K}^{(j)}$已知，以第一个相机确定世界坐标
- 为实现重建，观察点数需满足

$$2NM \geq 3N + 6(M-1) - 1$$

- 实际中有某些点在某些视角不可见的情况，则需要更多观察点

运动推断结构(Structure from Motion)

光束平差法(Bundle Adjustment)

- 最小化投影误差

$$\sum_{j=1}^{M}\sum_{i=1}^{N}\left\|\mathbf{p}_i^{(j)}-\hat{\mathbf{p}}_i^{(j)}\right\|^2$$

$$\sum_{j=1}^{M}\sum_{i=1}^{N}\left\|\mathbf{p}_i^{(j)}-\pi\left(\mathbf{K}^{(j)}\left[\mathbf{R}^{(j)}\ \middle|\ -\mathbf{R}^{(j)}\mathbf{T}^{(j)}\right]\mathbf{P}_i\right)\right\|^2$$

考虑可见性

$$\sum_{j=1}^{M}\sum_{i=1}^{N}v_i^{(j)}\left\|\mathbf{p}_i^{(j)}-\pi\left(\mathbf{K}^{(j)}\left[\mathbf{R}^{(j)}\ \middle|\ -\mathbf{R}^{(j)}\mathbf{T}^{(j)}\right]\mathbf{P}_i\right)\right\|^2$$

运动推断结构(Structure from Motion)

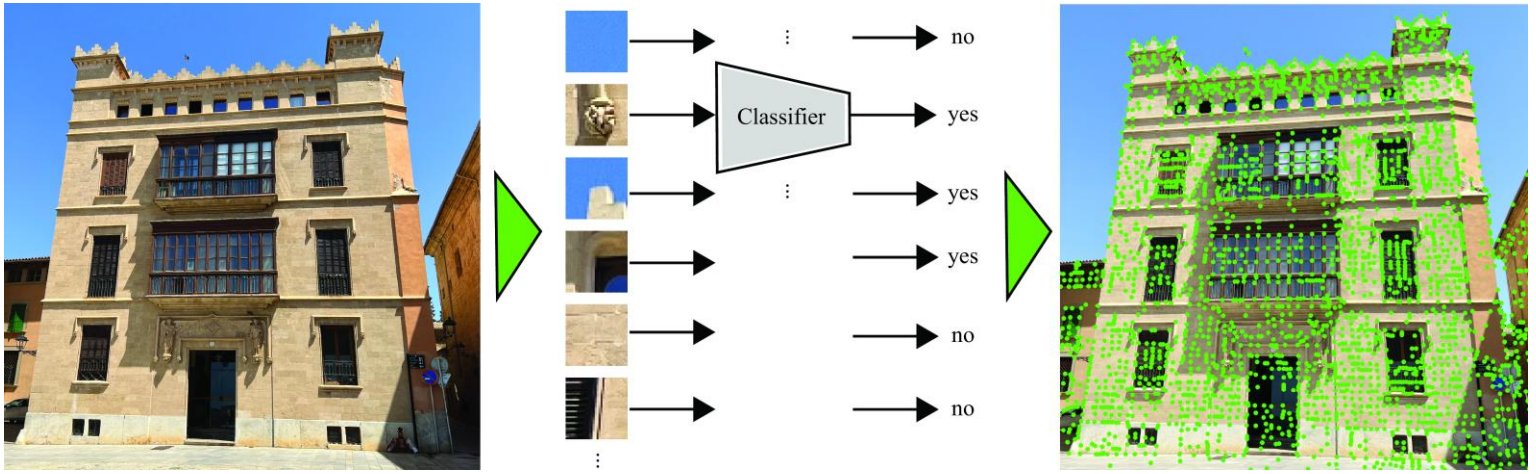
光束平差法(Bundle Adjustment)

$$\sum_{j=1}^{M}\sum_{i=1}^{N} v_i^{(j)} \left\| \mathbf{p}_i^{(j)} - \pi\left(\mathbf{K}^{(j)} \left[\mathbf{R}^{(j)} \mid -\mathbf{R}^{(j)}\mathbf{T}^{(j)} \right] \mathbf{P}_i \right) \right\|^2$$

- 非线性优化问题，局部最优
- 批量优化 （batch optimization）
- 渐进优化 （Progressive optimization）
  - ✓ 先以含匹配点最多的两张图进行重建
  - ✓ 逐渐添加新的相机

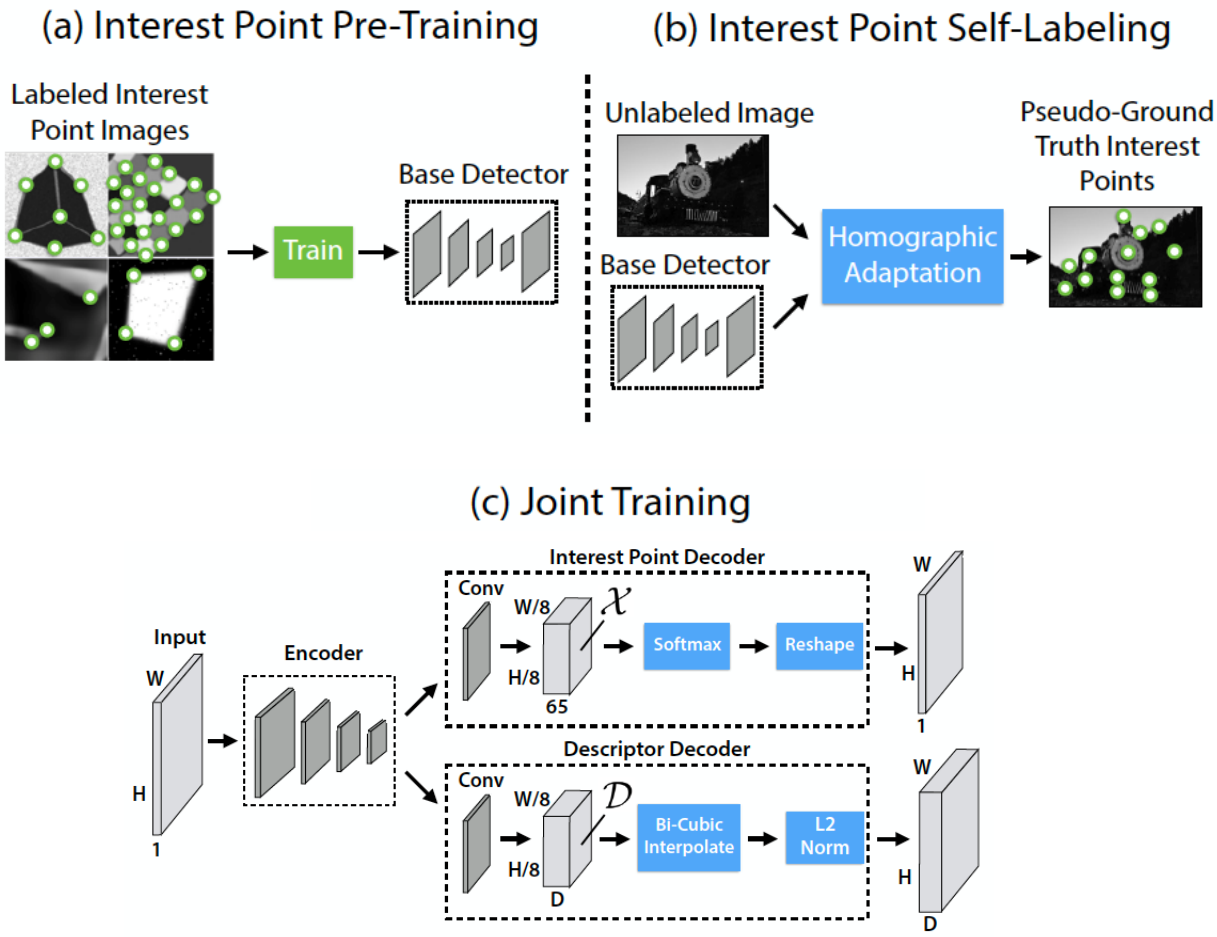
运动推断结构(Structure from Motion)

- 基于学习的关键点检测和局部特征表示
  - SuperPoint



运动推断结构(Structure from Motion)

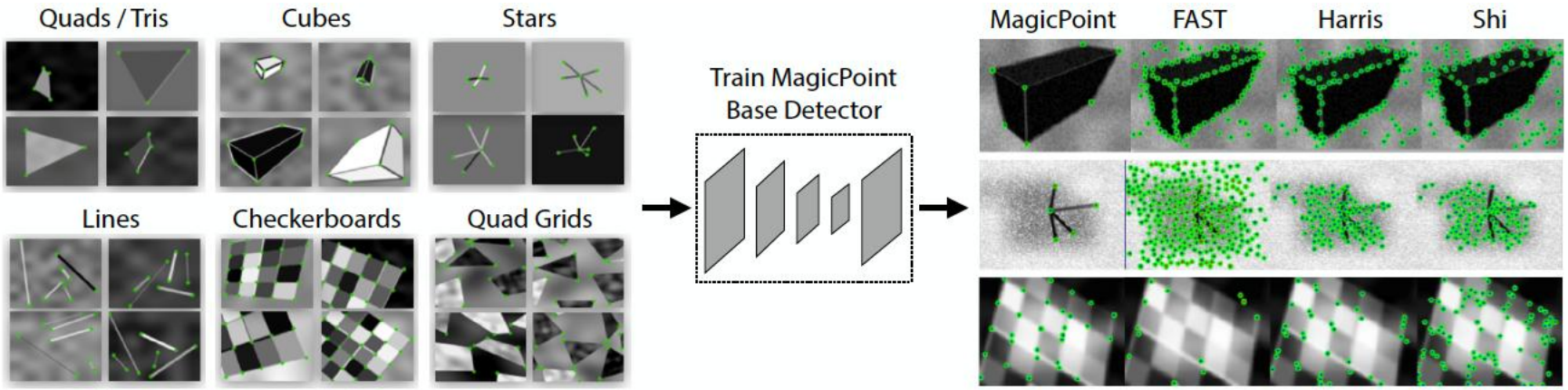
基于学习的关键点检测和局部特征表示

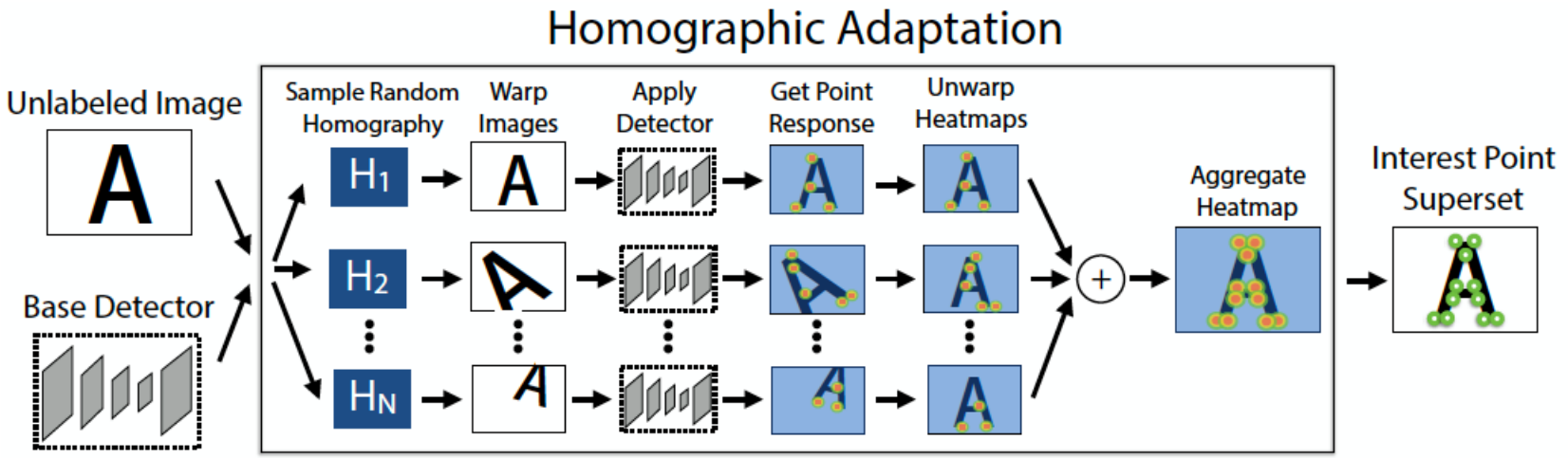


- D. DeTone, T. Malisiewicz, A. Rabinovich, SuperPoint: Self-supervised interest point detection and description. In CVPRW, 2018.

运动推断结构(Structure from Motion)

基于学习的关键点检测和局部特征表示

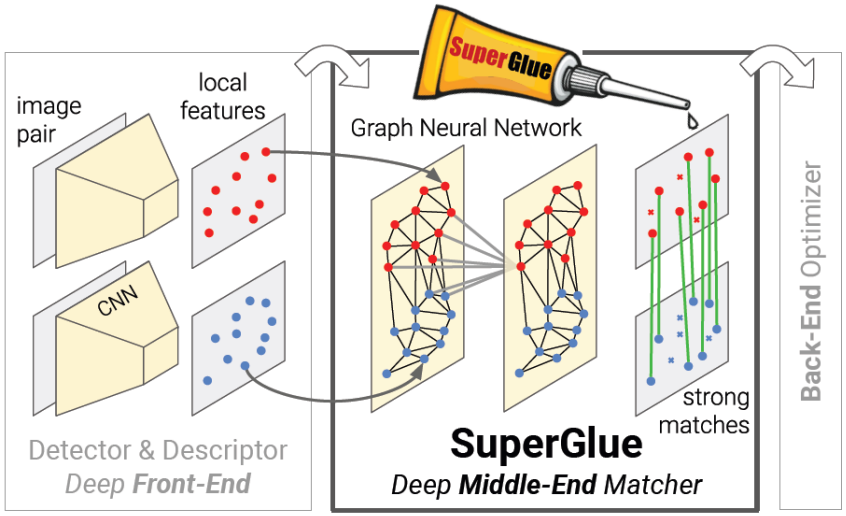


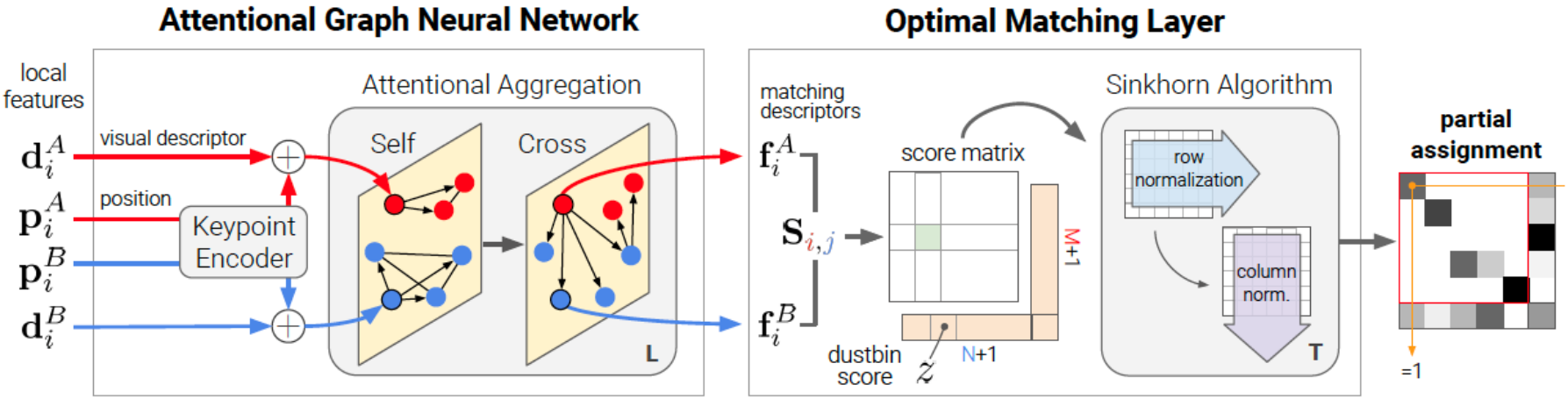


- D. DeTone, T. Malisiewicz, A. Rabinovich, SuperPoint: Self-supervised interest point detection and description. In CVPRW, 2018.

运动推断结构(Structure from Motion)

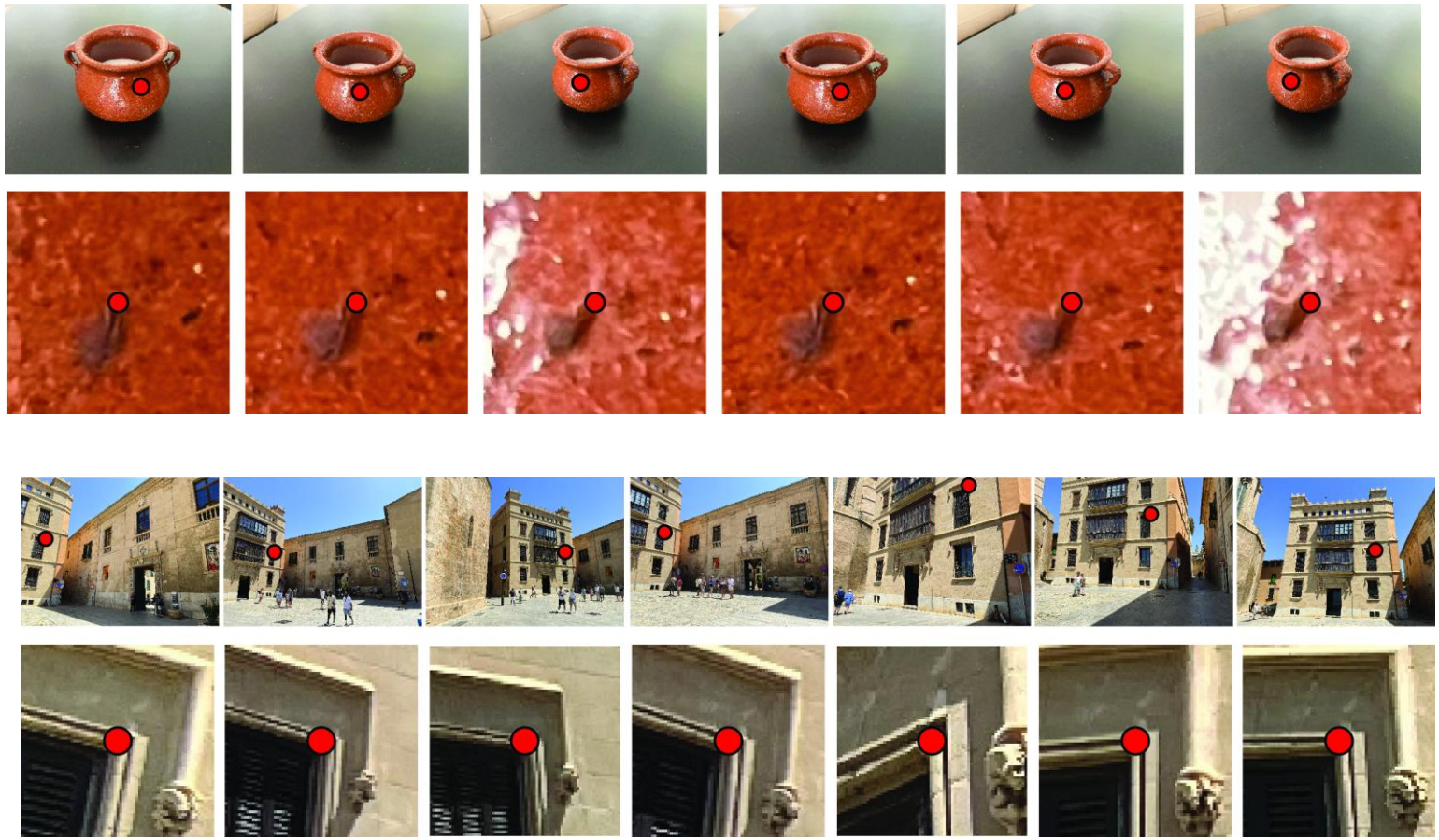
□ 基于学习的关键点匹配



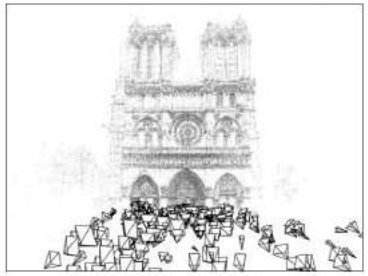


- P.-E. Sarlin, D. DeTone, T. Malisiewicz, A. Rabinovich, SuperGlue: Learning Feature Matching with Graph Neural Networks. In CVPR, 2020.

运动推断结构(Structure from Motion)



运动推断结构(Structure from Motion)





- N. Snavely, S. M. Seitz, R. SzeliskiPhoto, Tourism: Exploring Photo Collections in 3D. In SIGGRAPH, 2006.

3D图像分析

- 传统的三维重建方法
  - 成像变换和相机标定
  - 立体视觉和对极几何
  - 单应性
  - 运动推断结构
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  - 基于学习的单目深度估计
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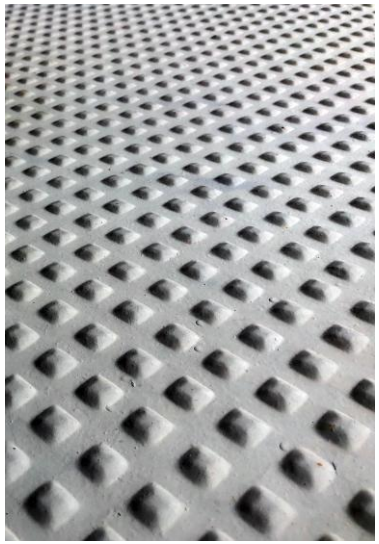
  - 基于多边形网格的三维表示
  - 基于隐函数的三维表示

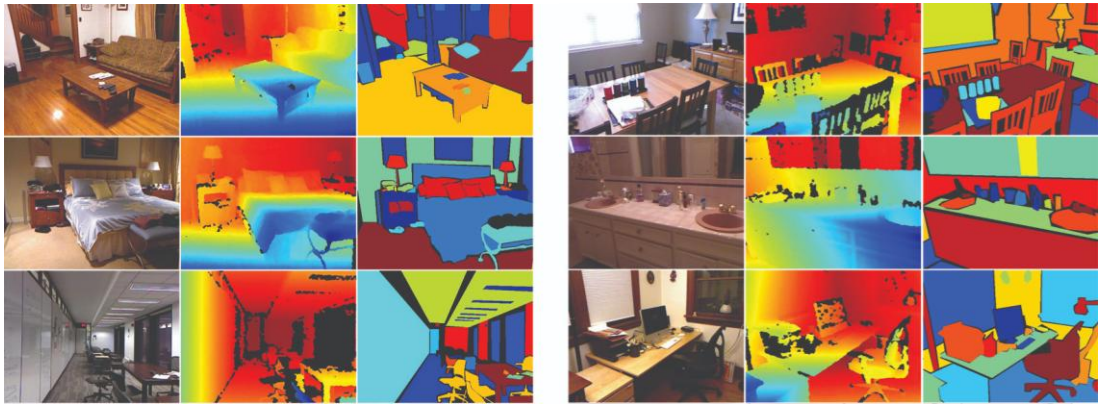
单目深度估计

- 图像中的深度线索 （So-called prior）
  - 影调(Shading): 表面朝向不同引起的亮度的空间变化
  - 纹理元尺寸、形状、空间关系等变化
  - 阴影和相互反射
  - 大气透视(Atmospheric perspective): 大气中光线的散射和吸收，使图像中的物体随距离增加而颜色变淡、对比减小、模糊
  - 相似物体

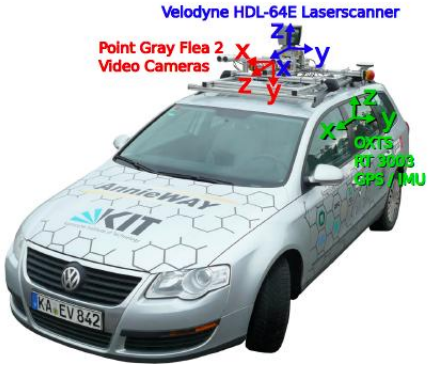


基于学习的单目深度估计

□ 数据采集



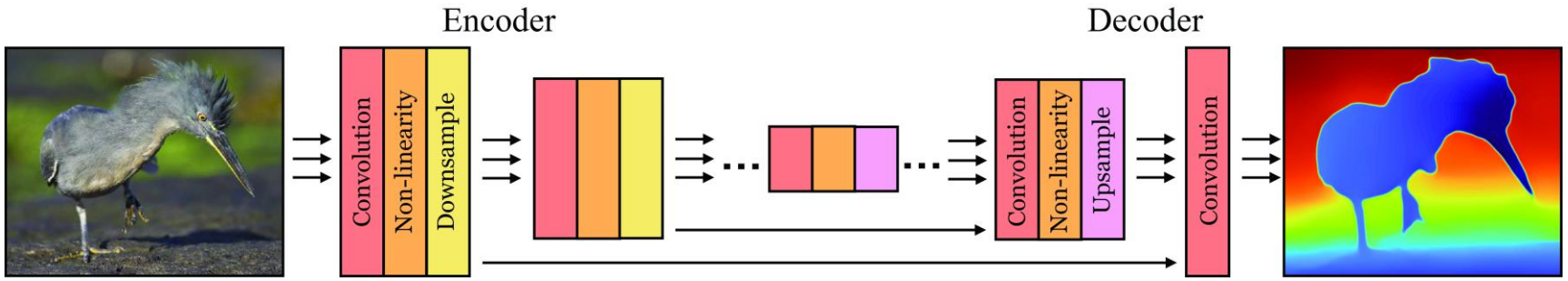
NYUv2 dataset

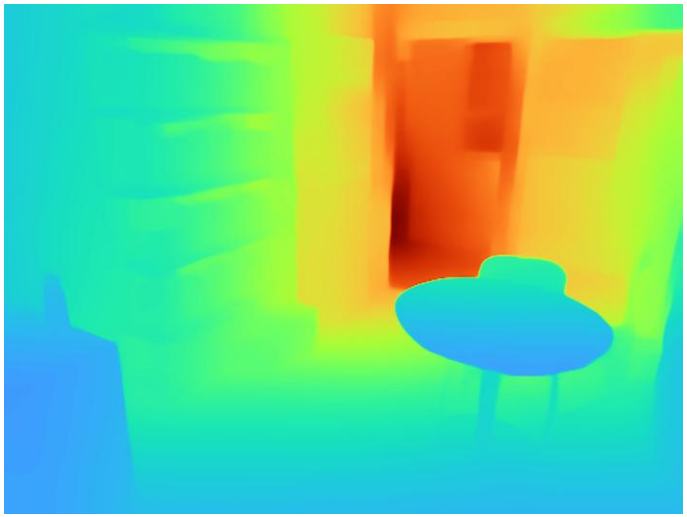


KITTI dataset

基于学习的单目深度估计

有监督学习

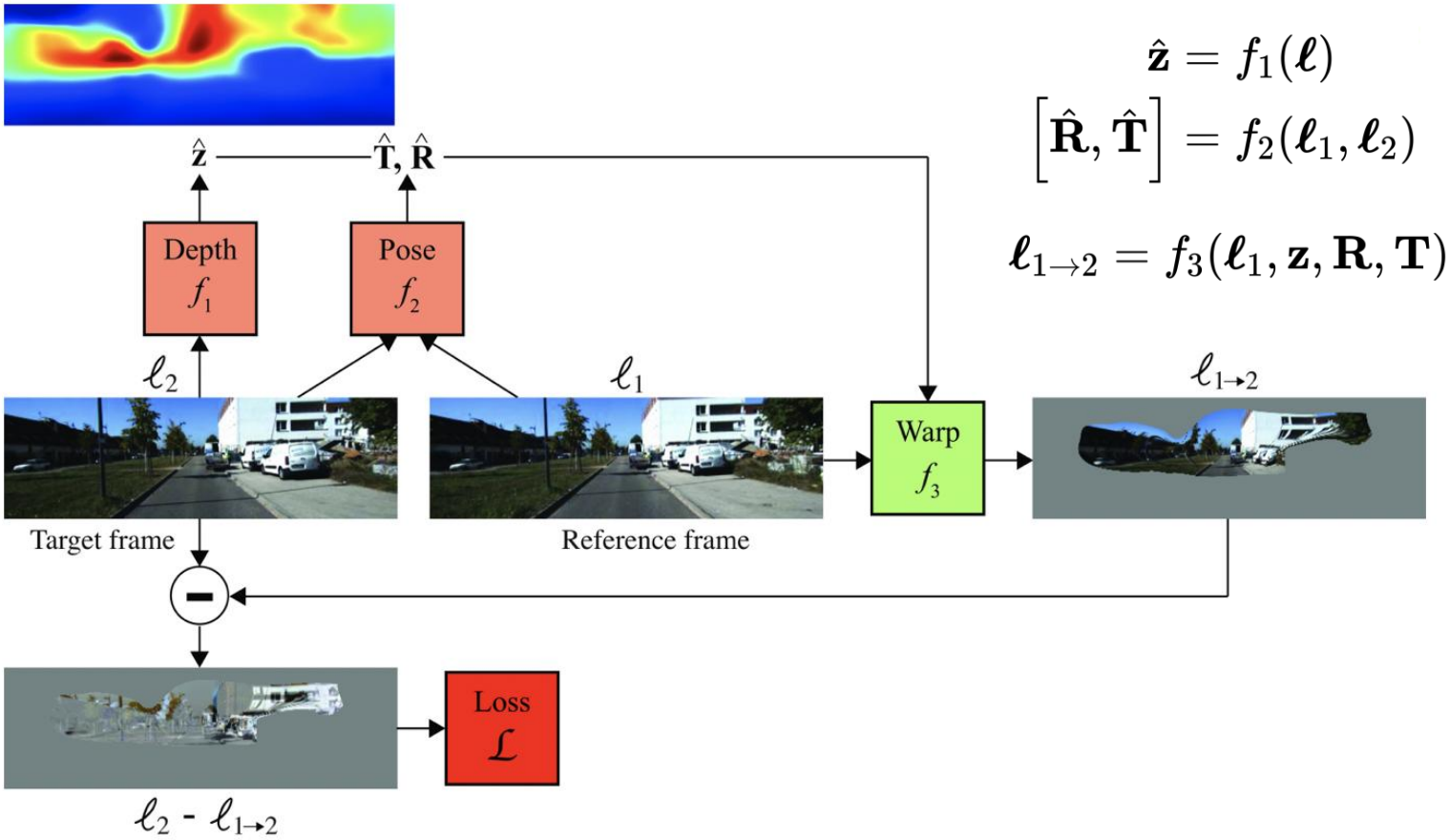




- K. Xian, C. Shen, Z. Cao, H. Lu, Y. Xiao, R. Li, Z. Luo, Monocular relative depth perception with web stereo data supervision., in: Cvpr, 2018: pp. 311–320

基于学习的单目深度估计

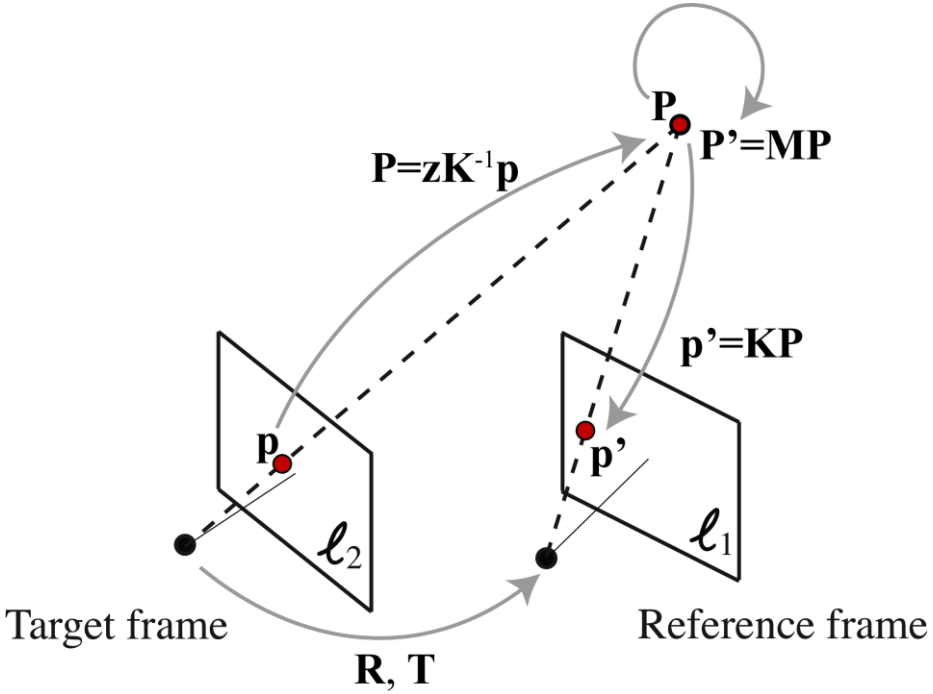
- 自监督学习 （videos or multiple frames）



- T. Zhou, M. Brown, N. Snavely, D.G. Lowe, Unsupervised learning of depth and ego-motion from video. In CVPR, 2017.

基于学习的单目深度估计

自监督学习



$$\ell_{1\to 2}(\mathbf{p}) = \ell_1\left(\mathbf{K}\hat{\mathbf{M}}\hat{z}(\mathbf{p})\mathbf{K}^{-1}\mathbf{p}\right)$$

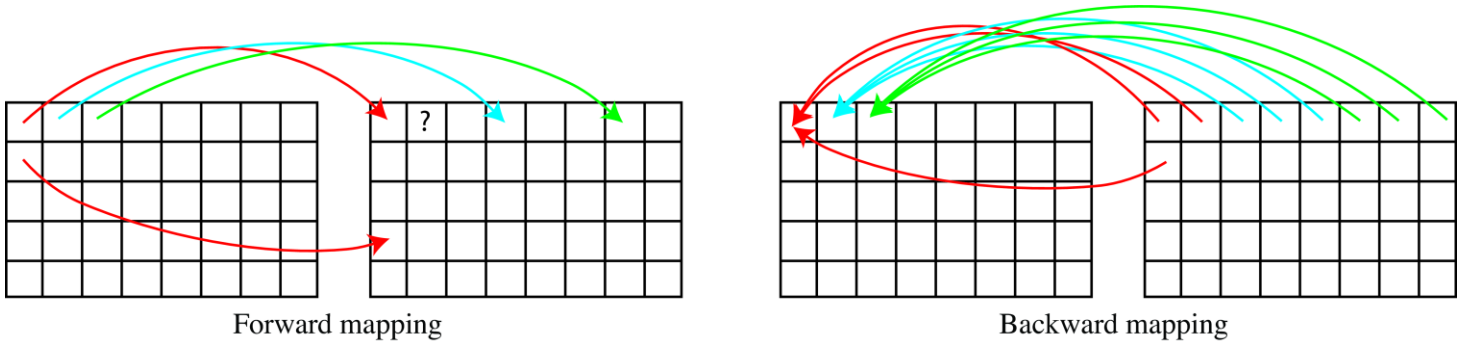
$$\mathcal{L} = \sum_{n,m} v(n,m)\left(\ell_{1\to 2}[n,m] - \ell_2[n,m]\right)^2$$

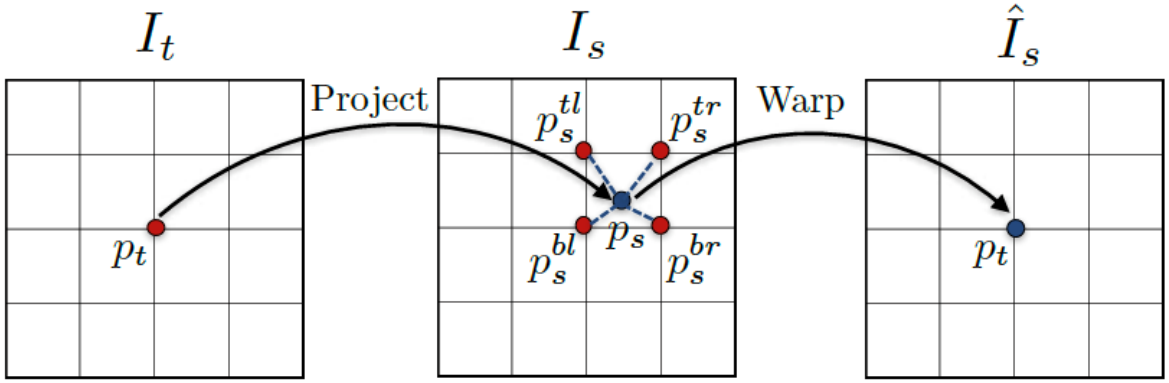
- T. Zhou, M. Brown, N. Snavely, D.G. Lowe, Unsupervised learning of depth and ego-motion from video. In CVPR, 2017.

基于学习的单目深度估计

自监督学习

- 图像变形：前向 vs. 后向

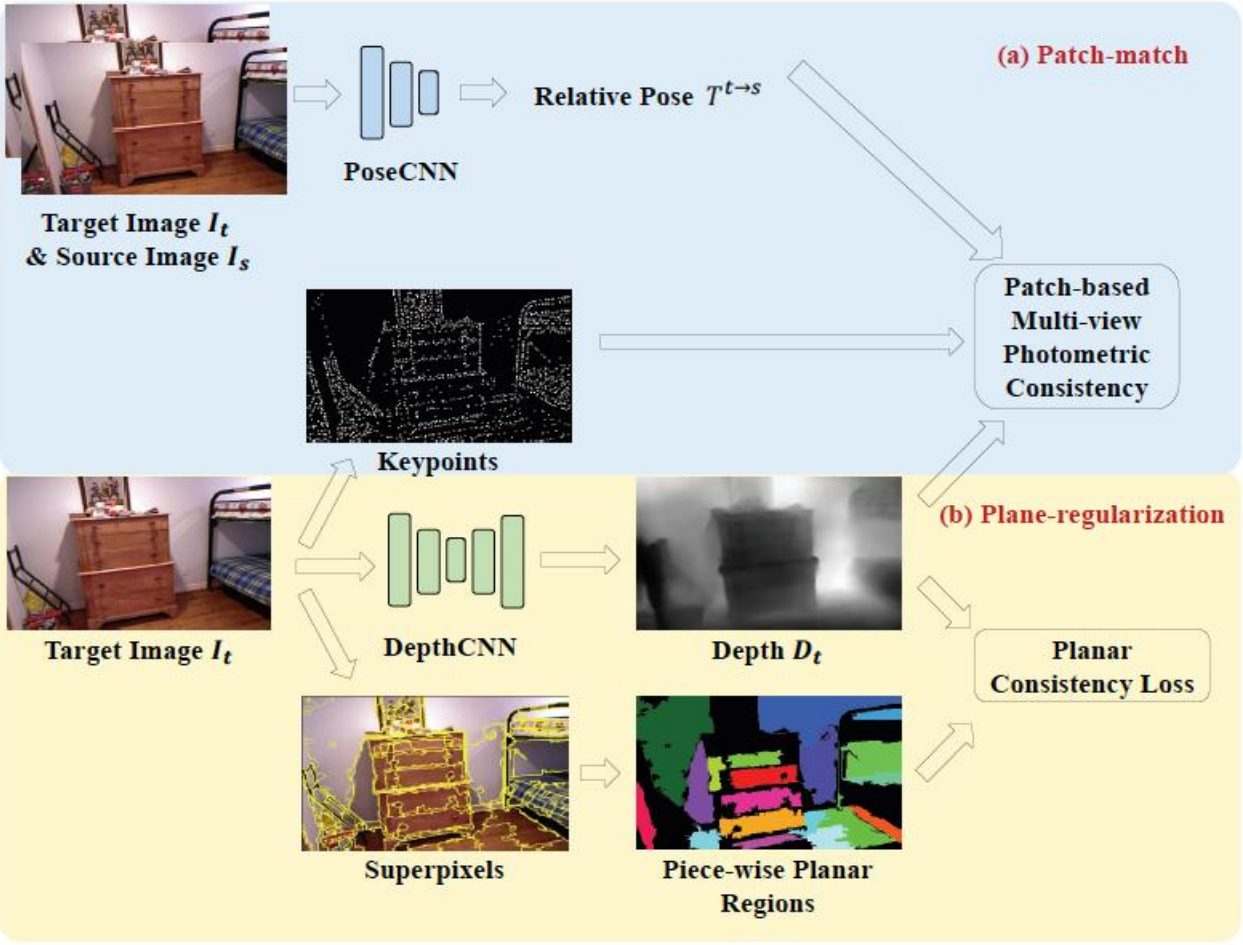




- T. Zhou, M. Brown, N. Snavely, D.G. Lowe, Unsupervised learning of depth and ego-motion from video. In CVPR, 2017.

基于学习的单目深度估计

□ 自监督学习

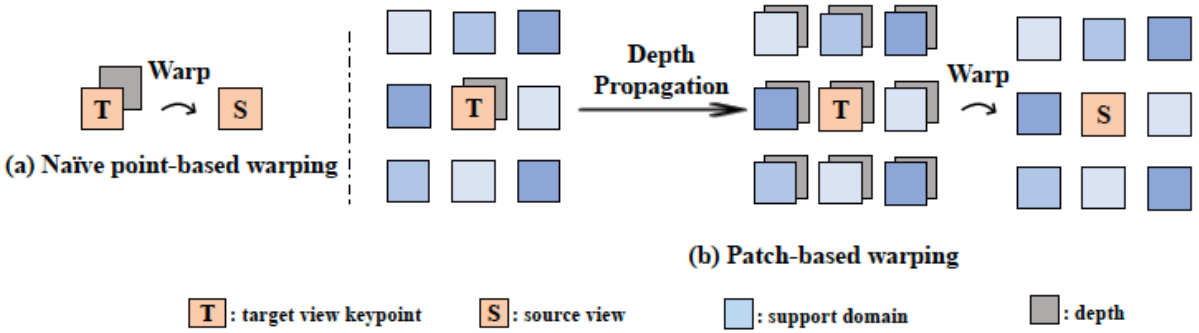


- Z.H. Yu, L. Jin, S.H. Gao, P2Net: Patch-match and Plane-regularization for Unsupervised Indoor Dept Estimation. In ECCV, 2020.

基于学习的单目深度估计

自监督学习

基于区块的多视角光度测试一致性



平面约束

$$p_n^{3D} = D(p_n)K^{-1}p_n, p_n \subseteq SPP_m$$

平面方程 $A_m^{\top}p_n^{3D} = \mathbf{1}$

$$L_{spp} = \sum_{m=1}^{M}\sum_{n=1}^{N}|D(p_n) - D'(p_n)|$$ 其中 $D'(p_n) = (A_m^{\top}K^{-1}p_n)^{-1}$

- Z.H. Yu, L. Jin, S.H. Gao, P2Net: Patch-match and Plane-regularization for Unsupervised Indoor Dept Estimation. In ECCV, 2020.